Physics I
ISI B.Math
Mid Semestral Exam : October 4, 2010

Total Marks: 70. Time: 2 and a half hours
Answer questions 1 and 2 and **any two** from questions 3, 4, and 5.

Question 1. Total Marks:15

A particle of mass m is subject to a one-dimensional force $\mathbf{F} = (-kx + bx^3)\hat{\mathbf{x}}$, where k and b are positive constants.

a.) Determine the potential energy as a function of position and plot it. [3]

b.) Determine the points of stable and unstable equilibrium and show them on the graph.[2]

c.) Suppose that the total energy E of the particle is given by $E = (3k^2)/(16b)$. Let the particle be initially at x=0 moving to the left. Show that the motion is bounded, determine the turning points and show them on the graph. [4]

d.) If the particle has the same energy as above but is found to at + infinity moving towards the origin, how close can it come to the origin? [3]

e.) What are the regions where the particle with the same energy as above is not allowed to be in by classical mechanics? [1]

f.) What are the possible motion of the particle if it has zero total energy [2]

Question 2. Total Marks: 15

a.) If the kinetic energy of a particle is decreasing between time t_1 and t_2, show that the angle between the acceleration and the velocity vector is always obtuse in the same interval. [3]

b.) Two stars are moving in each other's gravitational field. The mass of one star is twice the mass of the other. It is known that the trajectory of the relative position vector is a circle of radius r. Draw the trajectories of both the stars in the center of mass frame. Show clearly at least two positions of the two stars in their orbits. [4]

c.) A particle moving sweeps out 35 degrees arc of a circle as it moves from the farthest to the nearest point as measured from the source of a central force acting on the particle. What is the angle swept out as it moves from the nearest to the farthest position? Discuss if the orbit is periodic or not.[4]

d.) Sketch trajectory of a critically damped one dim. harmonic oscillator (assume that it is released from rest at x=A). If the same oscillator is also being driven by a large amplitude high frequency sinusoidal force, sketch the new trajectory and compare with the undriven case near t = 0 and $t = \infty$. [4]

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Question 3. Total Mark: 20

A fixed hollow sphere has center O and a smooth inner surface of radius b. A particle inside the sphere is projected horizontally from the lowest interior point with speed u. Let $\theta(t)$ be the angle the particle makes with the vertical radius at time t.

Derive the expression of speed of the particle in terms of u, g, b and θ.

Derive the expression of the Normal force exerted by the sphere on the particle in terms of m, u, g, b and θ where m is the mass of the particle.

Find the conditions on the initial speed so that the particle can perform complete circles.

How large the initial speed must be to satisfy these conditions?

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Question 4. Total Mark:20

A partially damped oscillator satisfies the equation $\ddot{x} + 2\kappa\dot{x} + \Omega^2 x = 0$.

Here Ω is a positive constant and κ is given by

$\kappa = 0$ for $x < 0$ and

$\kappa = K$ for $x > 0$.

where K is a positive constant and $K < \Omega$.

Find the period of the oscillator (as measured by the time interval between successive crossing the origin in the same direction) and the ratio of the successive maximum values of x.[Hint: Find solutions for $x > 0$ and $x < 0$ and combine them. Draw the graph of x vs. t.]

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Question 5. Total Mark:20

A spaceship of mass m is far far away ("at infinity") from an infinitely massive planet when all its propulsion engines fail. It has an initial speed V towards the planet and if there was no force between the planet and the spaceship, the spaceship would have missed the planet by a distance p. However, the plant is attracting the spaceship with a central force of magnitude $m\gamma/r^3$.

Show that if $\gamma = 8p^2V^2/9$, then the trajectory of the spaceship is given by

$$u = \frac{1}{r} = \frac{3}{p}\sin(\theta/3)$$

Plot the trajectory. Show that the spaceship will go around the plant once and then escape to infinity. Calculate the distance of closest approach and the speed of the aircraft at that instant. [Hint: What is the angular momentum of the spaceship? Use path equation and correct boundary conditions]